International Journal of Engineering Research (ISSN : 2319-6890)
Volume No.2, Issue No3, pp : 245-254 01 July 2013

A Review on ESPRIT-Estimation of Signal Parameters via Rotational Invariance Techniques

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Abstract:

High-resolution signal parameter estimation is a problem of significance in many signal processing applications. Such applications include direction-of-arrival (DOA) estimation, system identification, and time series analysis. A novel approach to the genera! Problem of signal parameter estimation is described. Although discussed in the context of direction-of-arrival estimation, FSPRIT can be applied to a wide variety of problems including accurate detection and estimation of sinusoids in noise. It exploits an underlying rotational invariance among signal subspaces induced by an array of sensors with a rotational invariance structure. The technique, when applicable, manifests significant performance and computational advantages over previous algorithms such as MEM, Capon's MLM, and Ml SIC.

Keywords: DOA, FSPRIT, MEM, MLM

1. INTRODUCTION

In many practical signal processing problems, the objective is to estimate from measurements a set of constant parameters upon which the received signals depend. For example, high-resolution direction-of-arrival (DOA) estimation is important in many sensor systems such as radar, sonar, electronic surveillance, and seismic exploration. High-resolution frequency estimation is important in numerous applications, recent examples of which include the design and control of robots and large flexible space structures. In such problems, the functional form of the underlying signals can often be assumed to be known (e.g. narrow-band plane waves, cisoids). The quantities to be estimated are parameters (e.g., frequencies and DOA's of plane waves, cisoid frequencies) upon which the sensor outputs depend, and these parameters are assumed to be constant.[1]

There have been several approaches to such problems including the so-called maximum likelihood (ML) method of Capon (1969) and Burg's (1967) maximum entropy (ME) method. Although often successful and widely used, these methods have certain fundamental limitations (especially bias and sensitivity in parameter estimates), largely because they use an incorrect model (eg. AR rather than special ARMA) of the measurements. Pisa renko (1973) was one of the first to exploit the structure of the data model, doing so in the context of estimation of parameters of cisoids in additive noise using a covariance approach. Schmidt (1977) and independently Bienvenu (1979) were the first to correctly exploit the measurement model in the case of sensor arrays of arbitrary form. Schmidt, in particular, accomplished this by first deriving a complete geometric solution in the absence of noise, then cleverly extending the geometric concepts to obtain a reasonable approximate solution in the presence of noise. The resulting algorithm was called MUSIC (Multiple Signal Classification) and has been widely studied. In a detailed evaluation based on thousands of simulations, M.I.T/s Lincoln Laboratory concluded that, among currently accepted high-resolution algorithms, MUSIC was the most promising and a leading candidate for further study and actual hardware implementation. However, although the performance advantages of MUSIC are substantial, they are achieved at a considerable cost in computation (searching over parameter space) and storage (of array calibration data).

In this paper, a new algorithm (ESPRIT) that dramatically reduces these computation and storage costs is presented. In the context of DOA estimation, the reductions are achieved by requiring that the sensor array possess a displacement invariance, i.e., sensors occur in matched pairs with identical displacement vectors. Fortunately, there are many practical problems in which these conditions are or can be satisfied. In addition to obtaining signal parameter estimates efficiently, optimal signal copy vectors for reconstructing the signals are elements of the ESPRIT solution as well. ESPRIT is also manifestly more robust (i.e., less sensitive) with respect to array imperfections than previous techniques including MUSIC.

To make the presentation as clear as possible, an attempt is made to adhere to a somewhat standard notational convention. Lowercase boldface italic characters will generally refer to vectors. Uppercase boldface italic characters will generally refer to matrices. For either real- or complex-valued matrices., $(\bullet)^*$ will be used to denote the Hermitian conjugate (or complex-conjugate transpose) operation. Eigenvalues of square Hermitian matrices are assumed to be ordered in decreasing magnitude,

as are the singular values of nonsquare matrices. Knowledge of the fundamental theorems of matrix algebra dealing with ei-gendecompositions and singular value decompositions (SVD) is assumed (cf. [2]).

THE DATA MODEL

Although FSPRIT is generally applicable to a wide variety of problems, for illustrative purposes the discussions herein focus on DOA estimation. In many practical signal processing applications, data from an array of sensors are collected, and the objective is to locate point sources assumed to be radiating energy that is detectable by the sensors (cf. Fig. 1). Mathematically, such problems are quite simply, although abstractly, modeled using Green's functions for the particular differential operator that describes the physics of radiation propagation from the sources to the sensors. For the intended applications, however, a few reasonable assumptions can be invoked to make the problem analytically tractable.

The transmission medium is assumed to be isotropic and nondispersive so that the radiation propagates in straight lines, and the sources are assumed to be in the far-field of the array. Consequently, the radiation impinging on the array is in the form of a sum of plane waves. For simplicity, it will initially be assumed that the problem is planar, thus reducing the location parameter space to a single-dimensional subset of (R, i.e., $\theta_i \in [-\pi, \pi]$, where θ_i is the direction-of-arrival (DOA) of the ith source.

The signals are assumed to be narrow-band processes, and can be considered to be sample functions of a stationary stochastic process or deterministic functions of time. This can represent a significant difference when estimating the signals themselves, leading to entirely different estimation algorithms. However, as far as estimation of signal parameters such as DOA is concerned, both as-sumptions lead to the same (suboptimal) algorithm under certain conditions (e.g., persistent excitation).

Since the narrow-band signals are assumed to have the same known center frequency[2] (ω_0), the ith signal can be written as $s_i(t) = u_i(t) \cos(\omega_0 t + v_i(t)$, where $u_i(t)$ and $v_i(t)$ are slowly varying[3] functions of time that define the amplitude and phase of $s_i(t)$, respectively. For such signals, it is often more convenient to use the complex envelope representation [3] in which s,(t) = Re {s(t)}, where $s(t) = u(t)\, e^{J(\omega 0t + vi(t)}$. Noting that the narrowband assumption implies $u(t) = u(t - \tau)$ and $v(t)= v(t - \tau)$ for all possible propagation delays τ, the effect of a time delay on the received waveforms is simply a phase shift, i.e., $s(t - \tau)= s(t)\, e^{-j\omega o\tau}$. The result is that $x_k(t)$, the complex signal output of the kth sensor at time t, can be written as

$$X_k(t) = \sum_{i=I}^{d} a_k(\theta_i)\, s_i(t - \tau_k(\theta_i))$$

$$= \sum_{i=I}^{d} a_k(\theta_i)\, s_i(t)_e{}^{-jw\tau_k(\theta_i)} \quad \textbf{(1)}$$

where $\tau_k(\theta)$ is the propagation delay between a reference point and the kth sensor for the ith wavefront impinging on the array from direction $\theta, a_k(\theta_i)$ is the corresponding sensor element complex response (gain and phase) at frequency ω_o, and there are assumed to be d point sources present.

Employing vector notation for the outputs of the m sensors, the data model becomes

$$x(t) = \sum_{i=I}^{d} a(\theta_i)\, s_i(t) \quad (2)$$

Where

$$a(\theta_i) = [a_1(\theta_i) e^{-j\omega_0 \tau i(\theta i)}), \ldots, a_m(\theta_i) e^{-j\omega 0 Tm(\theta i)}]^T \quad (3)$$

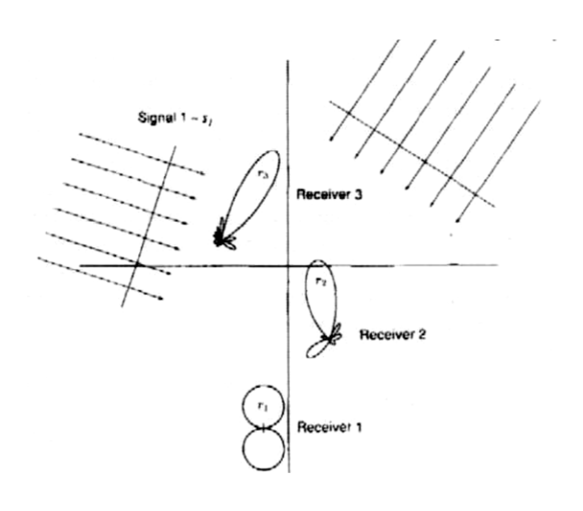
often termed the array response or array steering vector for direction θ_1. Setting $A(\theta) = [a(\theta_{1)}, \ldots \ldots a(\theta_d)]$, $s(t) = [s_1(t), \ldots, s_d(t)]^\tau$, and adding measurement noise n(t), the measurement model for the passive sensor array narrow-band signal processing problem is

$$x(t) = A(\theta)\, s(t) + n(t). \quad (4)$$

Note that $x(t), n(t) \in C^m$, $s(t) \in c^d$, and $A(\theta) \in C^{m\times d}$, and it will be assumed that $m > d$.

III. THE GEOMETRIC APPROACH

In 1977, Schmidt [4] developed the MUSIC (Multiple Signal Classification) algorithm by taking a geometric view of the signal parameter estimation problem. One of the major breakthroughs afforded by the MUSIC algorithm was the ability to handle arbitrary arrays of sensors.



Until the mid-1970's. direction finding

techniques required knowledge of the array directional sensitivity pattern in analytical form, and the task of the antenna designer was to build an array of antennas with a pre-specified sensitivity pattern. The work of Schmidt essentially relieved the designer from such constraints by exploiting the reduction in analytical complexity that could be achieved by calibrating the array. Thus, the highly nonlinear problem of calculating the array response to a signal from a given direction was reduced to that of measur-ing and storing the response. Although MUSIC did not mitigate the computational complexity of solution to the DOA estimation problem, it did extend the applicability of high-resolution DOA estimation to arbitrary arrays of sensors.

A. Array Manifolds and Signal Subspaces

To introduce the concepts of the array manifold and the signal subspace, recall the noise free data model x(t) = A(θ) s(t). The vectors a $(\theta_i) \in C'''$. the columns of A (θ) are elements of a set (not a subspace), termed the array manifold (a), composed of all array response (steering) vectors obtained as θ ranges over the entire parameter space. a is completely determined by the sensor directivity patterns and the array geometry, and can sometimes be computed analytically. However, for complex arrays that defy analytical description, a can be obtained by calibration (i.e., physical measurements).

For azimuth-only DOA estimation, the array manifold is a one-parameter manifold that can be viewed as a rope weaving through C'". For azimuth and elevation DOA estimation, the manifold is a sheet in C" To avoid ambiguities, it is necessary to assume that the map from θ = $\{\theta_{1.........}, \theta_d\}$ to $\Re$ {A(θ)}, the subspace spanned by the columns of A (θ), is one-to-one. This property can be ensured by proper array design.

The key observation is that if x(t) = a (θ) s_θ (t) is an appropriate data model (in the absence of noise) for a single signal, the data are confined to a one-dimensional sub-space of characterized by the vector a(θ). For d signals, the observed data vectors x(t) = A(θ) s(t) are constrained to the d-dimensional subspace of C'", termed the signal subspace (S_x), that is spanned by the d vectors $a(\theta_i)$, the columns of A(θ).

B. Intersections as Solutions

The concepts of an observed signal subspace and a calibrated array manifold permit an immediate visualization of the solution. In the absence of noise, the outputs of the sensor array lie in a d-dimensional subspace of C" the signal subspace (S_x) spanned by the columns of A(θ). Once d independent vectors have been observed, S_x is known, and intersections between the observed subspace and the array manifold yield the set of vectors from the array manifold that span the observed signal subspace. A three-sensor two-source example is graphically depicted in Fig. 2. Assuming that the sensor array has been designed such that the map from parameters to array manifold vectors is unique, the parameters are immediately determined.

Problems arise when only noisy measurements x(t) = A(θ) s(t) + n(t) of the array output are available, since S_x must be estimated- Imposing the constraint that the estimate $\hat{S}_x$ be spanned by elements from a and assuming unknown deterministic signals and Gaussian noise, a maximum-likelihood (ML) estimator can be formulated as described in Appendix A. However, the ML solution (of obtaining a set of vectors from the array manifold that best fits all the measurements) is computationally prohibitive in most practical applications.

Schmidt's idea was to employ a suboptimal two-step procedure instead. First, an unconstrained set of d vectors that best fits all the measurements is found. Then points of closest approach of the space spanned by those vectors to the array manifold are sought. This procedure, although clearly suboptimal. retains some of the key properties of the ML solution, including the fact that the exact answer is obtained asymptotically as the number of measurements goes to infinity.)

C. Estimating the Signal Subspace

To obtain an unconstrained estimate of the signal sub-space, the least-squares (LS) criterion is most often employed. The idea is to find a set of d vectors that span a subspace of C" that best fits, in an LS sense, the observed data. Assuming the signals and noise are zero-mean, a method can be derived by first examining the covariance matrix of the measurements. If the signals are modeled as stationary (zero-mean) stochastic processes, they arc assumed to be uncorrelated with the noise and possess a positive definite covariance matrix $R_{ss} > 0$. If, on the other hand, a deterministic (zero-mean) signal model is chosen, a persistent excitation condition is imposed, i.e.,

$$R_{ss} \overset{\text{def}}{=} \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} s(t)\, s*(t)$$

is assumed to exist and be positive definite.

Under the conditions given above, the covariance matrix of the measurements is given by

$$R_{xx} \overset{\text{def}}{=} E\{xx*\} = AR_{ss}A* + \sigma^2\Sigma_n \qquad (5)$$

The objective is to find a set of d linearly independent vectors that is contained in S_x = R{A}, the subspace of the eigenvectors have been proposed by several investigators, e.g., Johnson |6].

Although conceptually simple, the one-dimensional MUSIC measure has several drawbacks. Primarily, problems in the finite measurement case arise from the fact that since d signals are known to be present, d parameter estimates. $\{\theta_1...,\theta_d\}$ should be sought simultaneously by maximizing an

appropriate functional rather than obtaining estimates one at a time as is done in the search over $P_M(\theta)$. However, multidimensional searches are exponentially more expensive than one-dimensional searches. The price paid for the computational reduction achieved by employing a one-dimensional search for d parameters is that the method is finite-sample-biased in the multiple source environment (cf. [1]). Furthermore, in low SNR scenarios and in situations where even small sensor array errors are present, the ability of the conventional MUSIC spectrum to resolve closely spaced sources (i.e., observe multiple peaks in the measure) is severely degraded (cf. [1]). Nevertheless, it should be emphasized that in spite of these drawbacks, MUSIC has been shown to outperform previous techniques (cf. [7]).

Finally, as indicated above, MUSIC asymptotically yields unbiased parameter estimates since as the amount of data becomes infinite, errors in the estimate S_x vanish If the noise is spatially Gaussian and temporally independent, the distribution of the eigenvectors of the sample covariance is asymptotically Gaussian, with mean equal to the true eigenvectors (assuming distinct eigenvalues) and covariance that go to zero [9], [10]. Thus, the estimated signal subspace converges in mean-square to the true signal subspace, and the parameter estimates converge to the true values as well.

E. Summary of the MUSIC Algorithm

The following is a summary of the MUSIC algorithm based on the covariance approach described above.

1) Collect the data and estimate $R_{xx} \overset{\text{def}}{=} E\{xx^*\} = AR_{ss}A^* + \sigma^2\Sigma_n$ denoting the estimate R_{xx}.
2) Solve for the eigensystem; $R_{XX}\bar{E} = \Sigma_n\bar{E}\Lambda$, where is $\Lambda = \text{diag}\{\lambda_1, \bullet\bullet\bullet\ \lambda_m\}$, $\lambda_1 \ldots \lambda_m$, and $\bar{E} = [e_1.. |\ldots| e_m]$.
3) Estimate the number of sources d.
4) Evaluate

$$P_M(\theta)=\frac{a^*(\theta)a(\theta)}{a^*(\theta)E_N E_N^* a(\theta)},$$

where $E_N = \Sigma_n [\, e_{d+1}| \ldots - |e_m]$.

5) Find the d (largest) peaks of $P(\theta)$ to obtain estimates of the parameters.

An alternative to first forming the measurement covariance matrix and then performing an eigendecom posit ion is to operate directly on the measurements using the singular value decomposition (SVD). In addition to avoiding squaring the data, this approach has a nice geometric interpretation. Letting $X \in C^{m\times N}$ denote the data matrix, the objective is to obtain a set of vectors spanning the column space of the rank d matrix, $X \in C^{m\times N}$, that best approximates X in a least-squares sense. The solution is given by the d left singular vectors of X corresponding to the d largest singular values.[12]

It is easily demonstrated that the eigendecomposition and the SVD yield the same subspace estimate. If the SVD of $X/\sqrt{N}$ is given by $U\Sigma V^*$,

$$\frac{1}{N}XX^* = U\Sigma^2U^* = \check{R}xx.$$

since Σ is diagonal and real, and U and V are unitary. Thus, the left singular vectors, U, of X are the right eigenvectors of $\check{R}_{xx}$, the sample covariance matrix. Thus, in the absence of finite precision effects in computing the decompositions, the subspace estimates from both techniques are identical.

There are, however, significant computational differences. The computation of the full SVD of X is of order mN^2 which can be significantly larger than the $0(m^3)$ operations required for an eigendecomposition of $\check{R}xx$. The increase in computation is due to the fact that the full SVD is also obtaining a set of d vectors, the first d columns of $V \in^{N*N}$, that span the d-imensional subspace of N-dimensional space spanned by the d signal vectors, vectors in C^N whose components are the samples of the underlying signals.[13] If the information of the full SVD is not required, partial SVD algorithms that compute only the left singular vectors and singular values can be cm-ployed resulting in substantial computational savings [11].

If N is the number of measurements to be processed, and m is the number of elements in each sample vector, forming the sample covariance matrix requires on the order of Nm^2 operations. Eigendecompositions of matrices in $\Re^{m\times m}$ require on the order of $10m^3$ operations, whereas a standard SVD of $X \in \Re^{m\times N}$ requires $\approx 2Nm^2 + 4w^3$ (cf. [2, p. 175]) if only the singular values and left singular vectors are required. The computational effort is therefore on the same order for both the SVD and eigenvector approaches to reducing the measurements to the statistic E_s used by ESPRIT and MUSIC Again, the major advantage of the SVD over the eigendecomposition is that the measurements are processed directly without squaring them. Thus, numerical problems associated with ill-conditioned matrices are mitigated to some extent by using the SVD.

IV. ESPRIT

Although MUSIC was the first of the high-resolution algorithms to correctly exploit the underlying data model of narrow-band signals in additive noise, the algorithm has several limitations including the fact that complete knowledge of the array manifold is required, and that the search over parameter space is computationally very expensive. In this section, an approach (ESPRIT) to the signal parameter estimation problem that exploits sensor ar-

ray in variances is described. ESPRIT is similar to MUSIC in that it correctly exploits the underlying data model, while manifesting significant advantages over MUSIC as described in Section I.

To simplify the description of the basic ideas behind ESPRIT, much of the ensuing discussion is couched in terms of the problem of multiple source direction-of-arrival (DOA) estimation from data collected by an array of sensors. For simplicity, discussions deal only with single-dimensional parameter spaces, e.g., azimuth-only direction finding (DF) of far-field point sources, since the basic concepts are most easily visualized in such spaces. Narrow-band signals of known center frequency will be assumed. Recall that a DOA/DF problem is classified as narrow-band if the signal bandwidth is small compared to the inverse of the transit time of a wave front across the array, and the array response is not a function of frequency over the signal bandwidth. The generality of the fundamental concepts on which ESPRIT is based makes the extension to higher spatial dimensions and to signals containing multiple frequencies possible.

A. Array Geometry

ESPRIT retains most of the essential features of the arbitrary array of sensors, but achieves a significant reduction in computational complexity by imposing a constraint on the structure of the sensor array, a constraint most easily described by an example. Consider a planar array of arbitrary geometry composed of m sensor doublets as shown in Fig. 3. The elements in each doublet have identical sensitivity patterns and are translationally separated by a known constant displacement vector Δ. Other than the obvious requirement that each sensor have nonzero sensitivity in all directions of interest, the gain, phase, and polarization sensitivity of the elements in the doublet are arbitrary. Furthermore, there is no requirement that any of the doublets possess the same sensitivity patterns although, as discussed in [1] and [14], there are advantages to employing arrays with such characteristics.

B. The Data Model

Assume that there are $d \leq m$ narrow-band sources centered at frequency ω_0 and that the sources are located

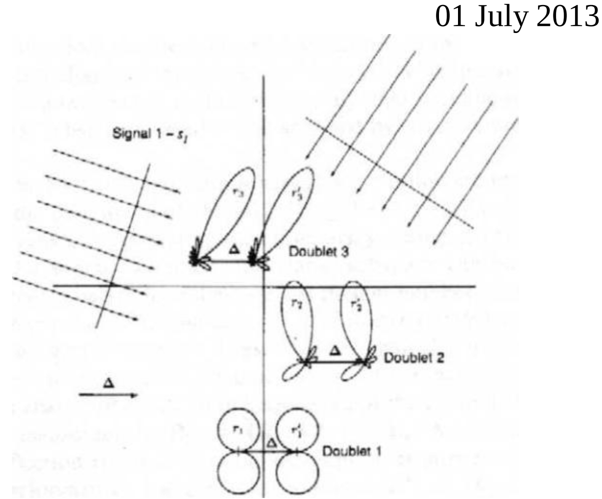


Fig. 3. Sensor array geometry for multiple source DOA estimation using ESPRIT.

sufficiently far from the array such that in homogeneous isotropic transmission media, the wavefronts impinging on the array are planar, As before, the sources may be assumed to be stationary zero-mean random processes or deterministic signals. Additive noise is present at all 2 m sensors and is assumed to be a stationary zero-mean random process with a spatial covariance $\sigma^2\Sigma_{n.}$

To describe mathematically the effect of the translational invariance of the sensor array, it is convenient to describe the array as being comprised of two subarrays, Z_x and Z_Y, identical in every respect although physically displaced (not rotated) from each other by a known displacement vector Δ of magnitude Δ. The signals received at the ith doublet can then be expressed as

$$X_i(t) = \sum_{k=1}^{d} s_k(t) a_i(\theta_k) + n_{xi}(t),$$

$$y_i(t) = \sum_{k=1}^{d} s_k(t) e^{j\omega 0 \Delta \sin \theta k / c} a_i(\theta_k) + n_{yi}(t),$$

where θ_k is now the direction-of-arrival of the kth source relative to the direction of the translational displacement vector Δ.

Since the sensor gain and phase patterns are arbitrary and since ESPRIT does not require any knowledge of the sensitivities, the subarray displacement vector Δ sets not only the scale for the problem, but the reference direction as well. The DOA estimates obtained are angles-of-arrival with respect to the direction of the vector Δ. A natural consequence of this fact is the necessity for a corresponding displacement vector for each dimension in which parameter estimates are desired.

Combining the outputs of each of the sensors in the two subarrays. the received data vectors can be written as follows:

$$x(t) = As(t) + n_x(t), \quad (9)$$

$$y(t) = A\,\Phi s(t) + n_s(r), \quad (10)$$

where the vector s(f) is the d x 1 vector of impinging

signals (wavefronts) as observed at the reference sensor of subarray Z_x. The signals can be correlated in the sense E $\{s_i (t) s^*_j (t)\} \neq 0$ for $i \neq j$ although the case of coherent (or fully correlated) sources is not considered herein (cf. [I, Section 7.11] for further discussion). The matrix Φ is a diagonal d × d matrix of the phase delays between the doublet sensors for the d wavefronts, and is given by

$$\Phi = \text{diag}\{e^{jy1}, \ldots, e^{jyd,}\}, \quad (11)$$

where $y_k = \omega\Delta \sin \theta_k/c$.Φ is a unitary matrix (operator) that relates the measurements from subarray Z_x to those from subarray Z_Y In the complex field, Φ is a simple scaling operator. However, it is isomorphic to the real two-dimensional rotation operator and is herein referred to as a rotation' operator. The unitary nature of Φ is a consequence of the narrow-band planewave assumption, an assumption that leads to unit-modulus eisoidal signals in the spatial domain. In time series analysis, the diagonal elements of Φ are potentially arbitrary complex numbers in which case Φ could be an expansive or contractive operator.

Defining the total array output vector as Z(t), the sub-array outputs can be combined to yield

$$z(t)=\begin{bmatrix} x\ (t) \\ y\ (t) \end{bmatrix} = \bar{A}s(t)+nz(t), \quad (12)$$

$$\bar{A}=\begin{bmatrix} A \\ A\Phi \end{bmatrix}, \ n_z(t)=\begin{bmatrix} n_z(t) \\ n_y(t) \end{bmatrix} \quad (13)$$

It is the structure of Ā that is exploited to obtain estimates of the diagonal elements of Φ without having to know A.

From (12), it is easily seen that the estimation problem posed is scale-invariant in the sense that absolute signal powers are not observable. For any nonsingular diagonal matrix, D, the data model is invariant with respect to the transformations $s(t) \rightarrow D^{-1} s(t)$ and $\bar{A} \rightarrow \bar{A}D$. Thus, estimates of the signals and the associated array manifold vectors derived herein are to be interpreted modulo an arbitrary scale factor unless knowledge of the gain pattern of one of the sensors is available

C. ESPRIT- The Invariance Approach

The basic idea behind ESPRIT is exploit the rotational invariance of the underlying signal subspace induced by the translational invariance of the sensor array. The relevant signal subspace is the one that contains the outputs from the two subarrays described above, Z_x and Z_Y. Simultaneous sampling of the output of the arrays leads to two sets of vectors, E_x and E_Y, that span the same signal subspace (ideally, that spanned by the columns of A).

The ESPRIT algorithm is based on the following results for the case in which the underlying 2m-dimensional signal subspace containing the entire array output is known in the absence of noise, the signal subspace can be obtained as before by collecting a sufficient number of measurements and finding any set of d linearly independent measurement vectors. These vectors span the d-dimensional subspace of C^m spanned by Ā. The signal subspace can also be obtained from knowledge of the covarianee of the measurements $R_{zz} = \bar{A}R_{SS}A^* \sigma^2\Sigma_n$. As discussed in Section III, if $d \leq m$ (an assumption required in later developments), the 2 m - d smallest GE's of (Rzz,Σ_n) are equal to σ^2. The GEV's corresponding to the d largest GE's are used to obtain $E_s = \Sigma_n[e_1| \ldots |e_d|$, where $\Re\{E_s\} = \Re\{\bar{A}\}$, the signal subspace.

Since $\Re\{E_s\} = \Re\{A\}$, there must exist a unique (re-call $d \leq m$), non singular T such that

$$E_S = \bar{A}\,T \quad (14)$$

Furthermore, the invariance structure of the array implies E_s can be decomposed into $E_x \in C^{m*d}$ and $E_Y \in C^{M*D}$ (cf. Z_y and Z_y subarrays) such that

$$E_S = \begin{bmatrix} E_X \\ E_Y \end{bmatrix} = \begin{bmatrix} AT \\ A\phi T \end{bmatrix} \quad (15)$$

from which it is easily seen that

$$\Re\{E_X\} = \Re\{E_y) = \Re\{A\}.$$

Since E_x and E_Y share a common column space, the rank of

$$E_{XY} \overset{def}{=} [E_x \mid E_Y] \quad (16)$$

is d, which implies there exists a unique (recall d < m) rank d matrix[18] $F \in C^{2d \times d}$ such that

$$0 = [E_X|E_Y|\ F = E_XF_X + E_YFy, \quad (17)$$

$$= A\ TF_X + A\Phi TF_Y \quad (18)$$

spans the null-space of $[\ E_x \setminus E_Y\backslash$. Defining

$$\psi \overset{def}{=} F_x[F_y]^{-1} \quad (19)$$

equation (18) can be rearranged to yield

$$AT\psi = A\Phi T \Rightarrow AT\psi T^{-1} = A\Phi.$$

Assuming A to be full rank implies

$$T\psi T^{-1} = \Phi \quad (21)$$

Therefore, the eigenvalues of ψ must be equal to the diagonal elements of Φ, and the columns of T are the eigenvectors of ψ This is the key relationship in the development of ESPRIT and its properties. The signal parameters are obtained as nonlinear functions of the eigen-values of the operator ψ that maps (rotates) one set of vectors [Ex) that span an m-dimensional signal subspace into another (Ey).

D. Estimating the Subspace Rotation Operator

In practical situations where only a finite number of noisy measurements are available, E_s is estimated from the covariancc matrices of the measurements R_{ZZ} or, equivalently, from the data matrix Z. The result is that $\Re\{E_S)$ is only an estimate of S_z, and with probability one, $\Re\{E_S\} \# \Re\{\bar{A}\}$. Furthermore, $\Re\{E_x) \# \Re\{E_Y\}$. Thus, the objective of

finding a ψ such that E_x ψ = E_y is no longer achievable. A criterion for obtaining a suitable estimate must be formulated. The most commonly employed criterion for problems of this nature is the least-squares (LS) criterion.

The standard LS criterion applied to the model AX = B to obtain an estimate of X assumes A is known and the error is to be attributed to B. Assuming the set of equations is overdetermined, the columns of A are linearly independent, and the noise in the elements of B is zero-mean and $E\{b_{ij}\ b_{kl}\} = \sigma^2\delta_{ik}\delta_{jl}$ the LS solution is

$\hat{X} = |A^*A|^{-1} A^*B$

It is easily verified that the estimate is unbiased and minimum variance. The extension to arbitrary, but known, covariance of the rows of B is straightforward and leads to the weighted least-squares (WLS) solution. If both A and B are noisy, however, the LS solution is known to be biased.

Since it is not difficult to argue that the estimates E_x and E_y are equally noisy, the LS criterion is clearly in appropriate. A criterion that takes into account noise on both A and B is the total least-squares (TLS) criterion. The TLS criterion can be stated [2| as finding residual matrices R_A and R_B of minimum Frobenius norm, and $\hat{X}$ such that

$$|A + R_A|\ X = B + R_B \quad (22)$$

This criterion is easily shown to be equivalent to replacing the zero matrix in (17) by a matrix of errors the Frobenius norm of which is to be minimized (i.e., total least-squared error). If the covariance of the errors, specifically the rows of $[R_A\ |R_B]$ is known to withm a scale factor, the TLS estimate of X is strongly consistent [8].

Appending a nontriviality constraint F*F =I to eliminate the zero solution and applying standard Lagrange techniques leads to a solution for F given by the eigenvectors corresponding to the d smallest eigenvalues of $E^*_{XY}E_{xy}$. The eigenvalues of ψ as defined above and cal culated from the estimates F_x and F_y are taken as estimates of the diagonal elements of Φ

E. Summary of the TLS ESPRIT Covariance Algorithm

The TLS ESPRIT algorithm based on a covariance formulation can be summarized as follows.

1) Obtain an estimate of R_{zz}, denoted $\hat{R}_{zz}$, from the measurements Z

2) Compute the generalized eigendecomposition of $\{R_{ZZ}, \Sigma_n\}$

$$R_{ZZ}\overline{E} = \Sigma_n \overline{E}\ \Lambda$$

where $\Lambda = \text{diag}\{\lambda_1,\dots \lambda_{2m}\}$, $\lambda_1 \geq \dots \geq \lambda_{2m}$, and $\overline{E} = |e_1|\dots|e_{2m}|$.

3) Estimate the number of sources $\hat{d}$.

4) Obtain the signal subspace estimate $\hat{S}_Z = \Re\{E_S\}$ and decompose it to obtain E_x and E_y, where

$$E_S \stackrel{\text{def}}{=} \sum\nolimits_n [e_1 | \dots | e_d] \Rightarrow \begin{bmatrix} E_X \\ E_Y \end{bmatrix}$$

5) Compute the eigendecomposition ($\lambda_1 > \dots > \lambda_{2d}$)

$$E^*_{XY}E_{XY} \stackrel{\text{def}}{=} \begin{bmatrix} E^*_X \\ E^*_Y \end{bmatrix} [E_X \mid E_Y] = E\Lambda E^*$$

and Partition E into $\hat{d} \times \hat{d}$ submatrices.

$$E \stackrel{\text{def}}{=} \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}$$

6) Calculate the eigenvalues of $\psi = -E_{12}\ E_{22}^{-1}$

7) Estimate $\hat{\theta}_k = f^{-1}(\hat{\phi}_k)$; $\hat{\phi}_k = \lambda_k\ (-E_{12}\ E_{22})^{-1}$ for all k = 1,2 $\hat{d}$ e.g., for DOA estimation, $\theta_k = \sin^{-1}\{c \arg(\phi_k\}/(\omega_0\Delta)\}$.

For arrays with multiple in variances, such as uniform linear arrays, the decomposition of E_s into Ex and E_Y is not unique. See [14| and [17] for more details concerning multiple invanance ESPRIT.

In many instances, it is preferable to avoid forming covariance matrices, and instead to operate directly on the data as discussed in Section 111. This approach leads to (generalized) singular value decompositions (GSVD's) of data matrices, and a GSVD variant of ESPRIT discussed in detail in [1].

From the key relation (21). several other quite striking results can be derived. For example, not only is knowledge of the array manifold not required, but the elements thereof associated with the estimated signal parameters (DOA's) can be estimated if desired. The same is true of the source correlation matrix, knowledge of which is not needed in ESPRIT.

E. Array Calibration

Using the TLS formulation of ESPRIT, the array manifold vectors associated with each signal (parameter) can be estimated (to within an arbitrary scale factor). From (21), the right eigenvectors of ψ are given by $E_\psi = T^{-1}$. This result can be used to obtain estimates of the array manifold vectors as

$$E_S E_\psi = \overline{A}\, TT^{-1} = \overline{A} \quad (23)$$

No assumption concerning the source covariance is required.

Although simple to compute, this estimate will not in general conform to the invariance structure of the array in the presence of noise. In low SNR scenarios, the deviation from the assumed structure $\overline{A} = [A^T\ |(A\Phi)^T]^T$ may be significant. In such situations, improved estimates of the array manifold vectors can be obtained by employing the formulation discussed in [18].

G. Signal Copy

International Journal of Engineering Research (ISSN : 2319-6890)
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In many practical applications, not only the signal parameters, but the signals themselves, are of interest. Estimation of the signals as a function of time from an estimated DOA is termed signal copy. The basic objective is to obtain estimates ˆs(t) of the signals s(t) from the array output, $z(t) = \bar{A} s(t) + n(t)$. Employing a linear estimator, a squared-error cost criterion in the metric of the noise (which is ML if the noise is Gaussian), and conditioning on knowledge of $\bar{A}$, leads to the estimate ˆs(t), the vector of coefficients resulting from the oblique projection of z(t) onto the space spanned by the columns of A (cf. the Appendix). The resulting weight matrix **W** (i.e., the linear estimator) whose ith column is a weight vector that can be used to obtain an estimate of the signal from the itha estimated DOA and reject those from the other

DOA's is given by

$$W = \sum\nolimits_n^{-1} \bar{A} \left[\bar{A}^* \sum\nolimits_n^{-1} \bar{A} \right]^{-1} \quad (24)$$

In terms of quantities already available, (24) can be written as

$$W = \sum\nolimits_N^{-1} E_S \left[E_S^* \sum\nolimits_N^{-1} E_S \right]^{-1} E_\psi^{-*} \quad (25)$$

using (23) to estimate Ā. This equivalence is easily established since from (21) it follows that the right eigenvectors of ψ equal T^{1}. Combining this fact with $E_S = \bar{A}T$ and substituting in (25) yields

$$W^* = E_\psi^{-1} \left[E_s^* \sum\nolimits_n^{-1} E_S \right]^{-1} E_S^* \sum\nolimits_n^{-1}$$

$$= \left[\bar{A}^* \sum\nolimits_n^{-1} \bar{A} \right]^{-1} \bar{A}^* \sum\nolimits_N^{-1} \quad (26)$$

Note that the optimal copy vector is a vector that is $\sum_n^{-1}$ orthogonal to all but one of the vectors in the columns of Ā since $W^* \bar{A} = I$

There is, of course, a total least-squares alternative to conditioning on knowledge of $\bar{A}$. Since only estimates of $\bar{A}$ are available, in low SNR scenarios where accurate signal estimates are desired, the TLS approach yields improved estimates at the cost of increased computation. Although not derived herein, ˆs{t) can be obtained by performing a (generalized) singular value decomposition of [$\bar{A}$ \z(t)|.] The right singular vector corresponding to the smallest singular value yields ˆs(t) as the first d elements after normalizing the last element to unity.""

H. Source Correlation Estimation

There are several approaches that can be used to estimate the source correlations. The most straightforward is to simply note that the optimal signal copy matrix W obtained above removes the spatial correlation in the observed measurements [cf. (25)]. Thus, $W^* C_{ZZ} W = DSD^*$ where S is the source correlation (not covariance) matrix, $C_{ZZ} = R_{ZZ} - \sigma^2 \sum_n$, and the diagonal factor D accounts for arbitrary normalization of the columns of W. Note that when R_{ZZ} must be estimated, a manifestly rank d estimate $\hat{C}_{zz} = E_S (\Lambda_s^{(d)} - \sigma^2 l_d | E_s^*$ can be used [cf. (6)], where $A_s^{(d)} = \text{diag}\{\lambda_1, \dots \lambda_d,\}$_and λ_i, is a GE of $(R_{zz}, \sum_n)$. Combining this with $E_S = A\,T$ gives

$$DSD^* = T[\Lambda_S^{(d)} - \sigma 2Id)T^*. \quad (27)$$

If a gain pattern for one of the elements is known, specifically if the gain $| a_{\{}(\theta_k)|$ is known for all θ_K associated with sources whose power is to be estimated, then source power estimation is possible since the array manifold vectors can now be obtained with proper scaling.

V. SIMULATION Results

Many simulations have been conducted exploring different aspects of ESPRIT and making comparisons to other techniques (cf. [1]). Herein, only one of the scenarios, but one that addresses several issues that arise in a practical implementation of ESPRIT, is presented. Thus, sensor gain and phase errors, as well as sensor spacing errors, are included. Furthermore, unequal source powers and a high degree of source correlation are assumed.

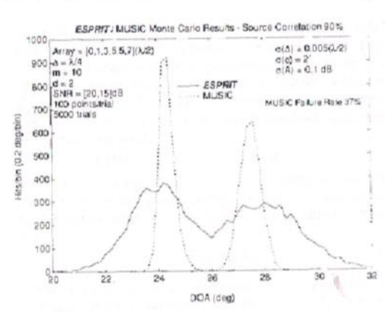
More specifically, the array chosen was a ten-element array with doublet spacing λ/4 and the five doublets randomly spaced on a line resulting in an aperture of approximately 4λ. Two sources were located at 24° and 28° (approximately 0.3 Rayleigh or 3 dB beamwidth separation), and were of unequal powers, 20 dB and 15 dB, respectively. Sensor errors were introduced by zero-mean normal random additive errors with sigmas of 0.1 dB in amplitude and 2° in phase[23] (independent of angle). Sensor location errors (along the axis of the array) with sigma 0.005 (λ/2) were included as well. The sources were 90 percent temporally correlated and 5000 trials were run. A histogram of the results is given in Fig. 4.

The number of sources was assumed to be known in the implementation of both MUSIC and ESPRIT. The indicated failure rate for MUSIC of 37 percent is the percentage of trials in which the conventional MUSIC spectrum did not exhibit two peaks in the interval [20°, 32° J. This, of course, is not an issue in ESPRIT, where two parameter estimates are obtained every time. The sample means and sigmas of the ESPRIT estimates were 23.93° ± 1.07° and 28.06° ± 1.37°, while those of the 3175 successful MUSIC trials were 24.35° ± 0.28° and 27.48° ± 0.38°, Note that with reference to Fig 4, there is an overlap in the distributions of the ESPRIT estimates. This has an effect on the statistics calculated, since a simple angle-ordering scheme was

International Journal of Engineering Research (ISSN : 2319-6890)
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used wherein the larger of the two angle estimates in each trial was associated with the 28° source. The effect is presumed to be small in this case.

The results indicate the presence of a bias even in the successful conventional MUSIC estimates, the source of which is described in detail in [1]. On the other hand, the ESPRIT estimates are unbiased, although of larger variance since less information concerning the array geometry is being utilized. Note also that in comparing the estimate variances, there is no attempt to account for the 1825 trials in which MUSIC failed[25] to provide two DOA estimates! However, as the subarray separation increases, the ESPRIT parameter estimate variances approach those of MUSIC. The same experiment was run for a subarray separation of 4λ, and the resulting ESPRIT estimates were 24.003° ± 0.062° and 28.002° ± 0.089°, and the corresponding MUSIC estimates were 24.011° ± 0.056° and 27.986° ± 0.078°. Due to the increased subarray spacing (array aperture), there were no MUSIC failures. Again. ESPRIT is unbiased, but now the sample parameter estimate sigmas are nearly equal to those obtained with MUSIC. More details concerning these and many other simulations can be found in [1] (see also



[18]).

VI Discussion

In this paper, a new technique (ESPRIT) for signal parameter estimation has been introduced. The algorithm differs from its predecessors in that a total least-squares rather than a standard least-squares (LS) criterion is employed. The earlier versions of ESPRIT described in [15] and [16] can be seen [1] to be least-squares estimators of an m x m operator whose action is restricted to a d-dimensional subspace. The fact that this LS operator is a restricted m x m operator leads to some concern over potential numerical difficulties in solving the generalized eigenproblem. Imposing the subspace restriction as a constraint prior to solving the generalized eigenproblem leads to a well-conditioned d x d generalized eigenproblem, thereby mitigating these numerical concerns; however, the least-squares property of the estimate is retained. In cases where the SNR is sufficiently large, the difference between the LS and TLS parameter estimates is small. The difference is notable at low SNRs, however; the LS estimates are biased as predicted, while the TLS estimates are relatively unbiased and have been shown |8] to be strongly consistent (converge with probability one to the true values).

A. ESPRIT and Null-Steering

Many of the previous high-resolution parameter estimation techniques are based on steering beams toward signal directions and. in some cases, simultaneously attempting to otherwise minimize the power in some weighted combination of sensor outputs. Parameter estimates are associated with DOA's at which peaks in the power output occur. Although intuitively appealing at first glance, there is a mueh more powerful alternative philosophy, that of null-steering. It is well known that deep, sharp notches in directivity patterns and filter gain functions are much easier to achieve than sharp peaks. Interferometers exploit this fact to obtain accurate estimates of source parameters by finding the relative phase required to cancel signal components in two channels. In this context. ESPRIT can be interpreted as a multidimensional null-steering parameter estimation algorithm. Calculation of the eigenvalues of the (rotation) operator $\boldsymbol{\psi}$ which are the roots of its characteristic polynomial, can be interpreted as multidimensional null-steenng. Instead of steering broad beams, ESPRIT steers sharp nulls at all sources simultaneously and does so without relying on knowledge of the array manifold!

B Computational Advantages of ESPRIT

The primary computational advantage of ESPRIT is that it eliminates the search procedure inherent in all previous methods (ML, ME, MUSIC). ESPRIT produces signal parameter estimates directly in terms of (generalized) eigenvalues. As noted previously, this involves computations of the order d^3. On the other hand, MUSIC and the other high-resolution techniques require a search over and it is this search that is computationally expensive. The significant computational advantage of ESPRIT becomes even more pronounced in multidimensional parameter estimation where the computational load grows linearly with dimension in ESPRIT, while that of MUSIC grows exponentially. If r_e is the resolution (i.e., number of vectors) required in the calibration of a for the L/th dimension in θ, the computation required to search over L dimensions for d parameter vectors is proportional to $\pi_l^L = 1^r{}_l\, r$,. For $r_{\{} = r$. the computational load is r^L.

APPENDIX

THE MAXIMUM LIKELIHOOD

Estimator For the class of problems considered herein, the maximum likelihood estimator is simple to derive analytically although, in most practical real-time applications, computationally prohibitive. For nonrandom signals in Gaussian noise with covariance $\sum_n$ z(t) = A(θ)s(t) + n(t), the likelihood function is easily written [51, {1);

$$\text{Log } [z(t)] = -\ln [P\{z(t)|z(t) = A(\theta)s(t) + n\}] \quad (28)$$

$$\propto - \{z(t) - A(\theta)\, s(t)]^*$$

$$\sum\nolimits_n^{-1} [z(t) - A(\theta)\, s(t)] \quad (29)$$

The maximization of Log is over {s (t); t∈ [o, N] } × {θ ∈ θ} and is therefore a non linear optimization problem. It belongs, however, to the class of separable non linear optimization problem. Golub and Pereyra [19] prove that the optimization can be carried out in two setps. A solution for the optimal s(t) is sought as a function of θ, then the maximization over θ is performed. Employing this procedure gives ˆs (t) = w* (θ) z (t). Where w (θ) = $\sum_n^{-1} A(\theta)\left[A^*(\theta)\sum_n^{-1} A(\theta)\right]^{-1}$. Substituting the expression for s (t) back into (29) and using standard properties of the trace operator,

$$\text{Log}(\theta) \propto -\text{Tr}\{P_{A\text{-}(\theta)} R_{ZZ} \textstyle\sum_n^{-1}\} \quad (30)$$

Where $P_{A\text{-}(\theta)}$ is the oblique projection operator into the complement of the space spanned by A(θ) (in the metric $\sum_n$). Maximization of this criterion is equivalent to finding.

$$\max_{\vartheta} T_r\{P_{A(\theta)} R_{ZZ} \textstyle\sum_n^{-1}\} \quad (31)$$

as can be easily verified. Although easy to describe analytically, the computational burden of actually carrying out the multidimensional projection and maximization over θ is generally prohibitive, resulting in the need for reasonable approximate solutions such as MUSIC and ESPRIT.

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